SCALE OF THE CONVECTION CELL STRUCTURE IN A HORIZONTAL LAYER OF LIQUID WITH UNSYMMETRICAL BOUNDARY CONDITIONS FOR HEAT TRANSFER

T. L. Perel'man and E. A. Romashko

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An interpretation is given of some experimental results on the effects of initial conditions on the cellular structure of convective motion.

Much detail is available about the conditions for convection in an unbounded horizontal layer of liquid; however, the details of this extremely interesting motion continue to attract experimental attention.

A recent Japanese study [1] deals with the effects of initial conditions on the parameters of the cellular structure in a horizontal layer of liquid; the top of the chamber, which was filled with silicone oil, was kept at a constant temperature. The heating was provided by infrared radiation coming through the mica base. It was found that a cellular structure with a cell size of about 2.3 units in terms of the layer thickness H was established if there was no perturbing grating under the base. If the initial circulation was characterized by parameters of 3, 2.4, or 2, then there was no major modification in the cellular structure after removing the perturbing grating. On the other hand, analogous experiments with initial cell parameters above 3.5 or less than 2 produced a change in the initial structure and formation of new cells with parameters from 2 to 2.5 when the grating was removed.

The following model is used in a mathematical interpretation of these results. If we assume that the heater temperature is considerably above the oil temperature, then the radiative heat transfer can be represented as a constant heat flux at the boundary, so we can assume that the conditions correspond to the generalized Rayleigh case for a horizontal layer of liquid bounded by solid plates, with boundary conditions of the first kind at the top and of the second kind at the bottom.

We handle the problem within the small-perturbation approximation of the theory of hydrodynamic stability.

A formulation of the problem can be found in [2]; here we merely note that the boundary conditions for the temperature perturbation amplitude may be put as

$$T=0 \text{ at } z=H, \quad \frac{dT}{dz}=0 \text{ at } z=0, \tag{1}$$

where z is the vertical coordinate with its origin at the lower surface, H is the height of the liquid layer, and T is the amplitude of the temperature perturbation.

It is possible to estimate [2] the critical value for the wave number M^* in this case; this should lie between the maximum value $M^* \sim 3.2$, which corresponds to zero boundary conditions for the temperature perturbation, and the maximum value $M^* \sim 1.2$ corresponding to zero boundary conditions for the derivative of the temperature perturbation. The corresponding wave numbers for the cell size in units of H are about 2 and 5.2.

A fuller study of the conditions for instability, and also of the possible steady-state motion (neutral perturbations) requires a solution to the corresponding problem for the eigenvalues for the boundary

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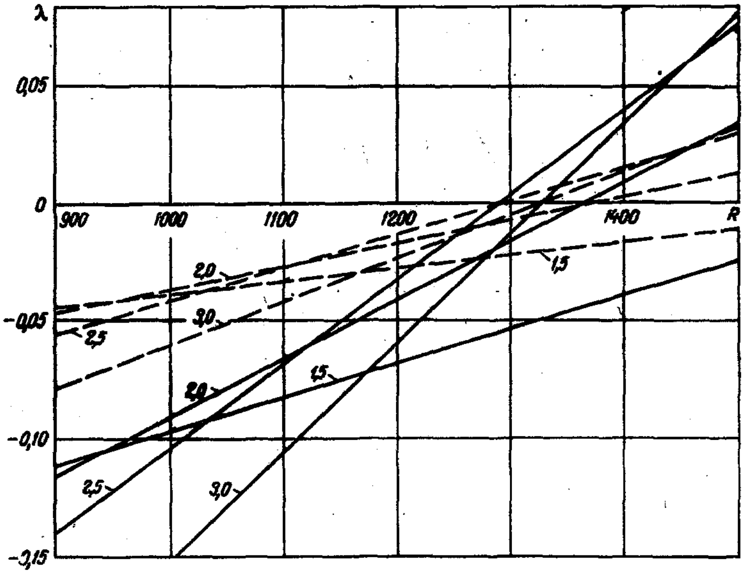

Fig. 1. First eigenvalue λ as a function of R for various M (numbers on curves) for Pr of 20 (solid lines) and 50 (broken lines).

conditions and temperature perturbation of (1). We use the numberical method of [3], and the following are some basic points from this.

We use a fully stabilized power-series method from linear algebra to construct an invariant subspace containing the first three eigenvectors for the problem, the approximate values for these vectors are determined from the corresponding finite-difference equations. The coordinate dependence of the approximations at each step in time is found via matrix methods. The characteristics for the neutral perturbations are determined by setting the first eigenvalue equal to zero.

A Minsk-22 computer was used in numerical realization.

The silicone oil had a melting point of −70°C and a boiling point of 282°C [4], so the convective instability is best discussed in the range 80-200°C, and for this purpose we made calculations for two values of the Prandtl number: Pr = 50 and Pr = 20. The temperature dependence of the latter was neglected. The nonuniformity in the parameters due to the temperature dependence usually has little effect on the stability and merely results in slight shift in the critical parameters (section 6 of [5]).

Figure 1 shows results for the first eigenvalue λ_1 in relation to the Rayleigh number R for various M; it is clear that λ_1 is linearly dependent on R within the range of the latter, and also that R is not dependent on the Prandtl number for neutral perturbations (the points of intersection of the graphs with the horizontal line $\lambda_1 = 0$). The latter result arises because the initial equations did not incorporate the temperature dependence of the viscosity.

The second and third eigenvalues are negative in the above range in Rayleigh number; the following are values for R = 100 and M = 2.5:

$$\lambda_1 = -0.042,\ \lambda_2 = -0.517,\ \lambda_3 = -1.348 \text{ for } \Pr = 50;$$

$$\lambda_1 = -0.105,\ \lambda_2 = -1.291,\ \lambda_3 = -3.367 \text{ for } \Pr = 20.$$

TABLE 1. Values of R in Relation to M for Neutral Perturbations with Step Sizes h in Finite-Difference Approximation for Pr = 20

h	*M*			
	2,50	2,55	2,60	2,65
0,01	1290,5	1290,0	1290,4	1291,9
0,00625	1293,0	1292,4	1292,9	1294,3
0,005	1293,8	1293,1	1293,4	1295,0
0,003448	1294,3	1294,0	1294,7	1296,4

The $R = f(M)$ curve for neutral perturbations was calculated with $h = 0.01$ for the step in the finite-difference approximation; the region of the minimum on this curve was calculated with various values for h (Table 1). The critical values are correspondingly $R^* = 1294$, $M^* = 2.57$; the latter result gives the horizontal scale L of the cellular structure as $L = 2\pi/M^* \approx 2.44$, which in general agrees well with the above experimental results.

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